On the Mechanics of some Replica Bows. [1]

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1 Introduction

In this paper different approaches to the study of ancient bows are compared. One way of studying ancient bows is to make replica bows for experiments, see Bergman, McEwen & Miller[1–3] and Alrune[4]. Another way is to make mathematical models. Such models enable us to compare the performance of bows used in the past and in our time. Part of the modelling process is to identify all quantities which determine the action of the bow and arrow combination. Calculations are possible only when all the so-called design parameters are known. Descriptions of bows in the literature are often incomplete. For instance artistic representations give a limited amount of information. In other cases parts of recovered bows are missing. Often the researchers do not know all the design parameters and are not aware of their importance for a good understanding of the features of the bow at hand. In a former paper, Kooi[5], we summarized all the important quantities which determine the mechanical action of a bow. In Table 1 the most important parameters are recalled and a nomenclature is introduced.

Table 1: The most important design parameters of a bow and arrow combination and quality coefficients which measure the performance.

symbol	unit	description
$\overline{L}$	cm	half length of the bow
$\overline{m}_a$	kg	half mass of the arrow
$\lvert\overline{OD}\rvert$	cm	draw
$\overline{F}(\lvert\overline{OD}\rvert)$	kgf	weight
$\overline{m}_b$	kg	half mass of the bow
$\overline{A}_b$	kgf cm	amount of elastic energy in fully drawn bow
$\overline{\delta}_{bv}$	kgf cm/kg	energy storage capacity per unit of mass
a_D	—	utility coefficient
q	—	static quality coefficient
η	—	efficiency
$\overline{c}_l$	cm/s	initial velocity of the arrow

Both approaches supplement each other. Simplifications are necessary to keep mathematical models manageable. Therefore these models have to be validated by comparison

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of predicted results with actually measured quantities. Validation is a very difficult and complex subject. To take full advantage of modelling one does not want to check the model for all possible parameter combinations. So, a limited number of characteristic situations is chosen for which the comparisons are made. If the observed differences are small enough the model is also assumed to hold in other situations. The required degree of agreement between model predictions and measured data is often determined by the experience of the researcher and is by no way objective in practice. For validation purposes replica bows can be employed. On the other hand even simple mathematical models can be of help with the production of test procedures and the design of experimental set-up.

2 Description of the replica bows

We consider a number of bows used in an experimental study with replica bows of a variety of types in Bergman *et al.*[2]. These bows are: a replica of a Medieval longbow, a replica of an Egyptian composite bow and a replica of a Tartar bow.

Furthermore we consider a replica of a mesolithic Elm bow described by Alrune[4]. The bow is the Holmegaard bow 7000-7400 B.C. This bow is a flat bow with length $2\overline{L} = 154$ cm.

We assume that the replica bows each resemble one of the 'theoretical' bows described in Ref.[6] and which represent different type of bows: longbow $\Leftrightarrow$ KL-bow (non-recurve bow, Figure 1(a) in Ref.[5]), Egyptian angular bow $\Leftrightarrow$ AN-bow (non-recurve bow, Figure 1(b) in Ref.[5]) and Tartar bow $\Leftrightarrow$ TU-bow (static-recurve bow, Figure 2(b) in Ref.[5]). The half length of the Holmegaard bow is 1.166 times the draw being 66 cm. Therefore we assume that it can be represented by a KL-bow with length $\overline{L} = 1.143 \cdot |\overline{OD}|$, see Kooi[6]. This bow is denoted as the HO-bow.

Finally we study a modern working-recurve bow. We refer to Tuijn & Kooi[7] for a description of the bow, the test set-up and the experimental procedures. In Kooi[8] the mathematical model of this bow, the WR-bow, is described.

For the replica bows described in Ref.[2] the masses of the bows are known (McEwen, personal communication) and this makes it possible to estimate the effectiveness of the usage of the materials of the limbs. The overall length $2\overline{L}$, the weight $\overline{F}(|\overline{OD}|)$ and length $|\overline{OD}|$ as well as the mass of these bows $2\overline{m}_b$ are provided in Tabel 2. In Ref.[5, Tabel 2] we gave the mechanical properties and the energy storage capacity per unit of mass for materials used in making bows. This data can be used to estimate the maximum amount of energy which can be stored in the fully drawn bow.

The actually stored elastic energy in the limbs of the fully drawn bow is denoted by $\overline{A}_b$. It is calculated using the mathematical model whereby the actual values for the draw and weight are used. The quotient of actually stored energy per unit of mass of the limbs in the fully drawn bow $\overline{A}_b/2\overline{m}_b$ and the maximum allowable energy per unit of mass $\overline{\delta}_{bv}$ is denoted as the utility coefficient a_D. This quantity equals 1 when all material is used to the full extent. In practice it is smaller. We conclude that the materials of the replica bows are used rather well. Observe that the value of $\overline{\delta}_{bv}$ for the composite Egyptian and

Table 2: Measured data for replica bows and an estimation of utility coefficient a_D. The amount of energy $\overline{A}_b$ in the fully drawn bow is calculated using the mathematical model.

Ref.	Bow type		$\overline{F}(\lvert\overline{OD}\rvert)$ kgf	$\lvert\overline{OD}\rvert$ cm	$2\overline{m}_b$ kg	$2\overline{L}$ cm	$\overline{\delta}_{bv}$ kgf cm/kg	$\overline{A}_b$ kgf cm	a_D
[2]	longbow	KL	36.2	81.3	0.660	193	9000	1530	0.26
[2]	Egyptian	AN	28.8	101.6	0.485	153	20000	1610	0.17
[2]	Tartar	TU	27.2	81.3	0.652	150	20000	2430	0.19

Tartar bow is taken as an average of the values for the materials of which it is made; horn, sinew and wood.

The utility coefficient a_D is slightly smaller for the composite bows than for the yew longbow but the product $a_D \cdot \overline{\delta}_{bv}$ is still higher. This shows that in the fully drawn situation these bows store more deformation energy per unit of mass than the longbow.

3 Comparison of the test results with predictions

In Tabel 3 the experimental data is shown for the bows each shooting a number of arrows with different masses. The product $q \cdot \eta$ is calculated with experimental data only. The quantity q measures the amount of mechanical energy stored in the fully drawn bow divided by the product of the weight and the draw. This quality coefficient is large for static-recurve bows because of the leverage action of the rigid ears. Observe that the amount of energy available for the acceleration of the arrow $q \cdot |\overline{OD}| \cdot \overline{F}(|\overline{OD}|)$ equals the amount of energy in the fully drawn bow $\overline{A}_b$ minus the amount of energy stored in the braced bow. The quantity η is the efficiency and this is defined as the part of the available energy transformed into kinetic energy of the arrow. Hence, the product $q \cdot \eta$ equals the amount of kinetic energy of the arrow per weight and per draw of the bow. small

Also the values for the modern working-recurve bow are shown. On all mentioned bows, excluding the replica of the Holmegaard bow, Dacron bowstrings were used. The mass of these strings were 12, 10, 15 and 6 gram for the longbow, Egyptian, Tartar and modern bow, respectively.

The results suggest that the initial velocity of the modern bow is large when the small weight of the bow is taken into account. The velocity of the 25 gram arrow shot with the Tartar composite bow is larger than when shot with the modern bow, but the weight of the modern bow is much smaller and its efficiency is obviously much larger, $q \cdot \eta = 0.21$ for the Tartar composite bow and $q \cdot \eta = 0.28$ for the modern working recurve bow. When the light 18 grams arrow is shot with the modern bow its velocity equals that of the 25 grams

Table 3: Measured data for a number of bows used in experiments and an estimation of the product $q \cdot \eta$ for different values of arrow masses; $m_a = \overline{m}_a/\overline{m}_b$.

Ref.	Bow type	$\overline{F}(\lvert\overline{OD}\rvert)$ kgf	$\lvert\overline{OD}\rvert$ cm	$2\overline{m}_a$ kg	$\overline{c}_l$ cm/s	$q \cdot \eta$	m_a
2	Medieval longbow	36.2	81.3	0.090	4300	0.29	.136
2				0.050	5300	0.24	.076
2	Egyptian composite	28.8	101.6	0.090	3200	0.16	.186
2				0.050	4000	0.14	.104
2				0.025	5200	0.12	.052
2	Tartar composite	27.2	81.3	0.050	5100	0.30	.076
2				0.025	6000	0.21	.038
7	modern composite	12.6	71.1	0.029	4200	0.29	.100
7				0.024	4550	0.28	.082
7				0.018	5000	0.26	.062
4	Holmegaard bow	29.5	66	0.030	4700	0.17	—

arrow shot with the Tartar composite bow and the efficiency is still better, $q \cdot \eta = 0.26$. This shows the influence of the superior performance of the modern materials, a good cast or fast arrow is combined with a high efficiency.

These measured values can be compared with calculated values for the different type of bows. The predictions are obtained with computer simulations by the use of the mathematical model described in Ref.[6] and[8]. Note that the mentioned bows are not modelled based on the replica bows. This means that we did not use the dimensions of the replica bows. The comparison has therefore to be crude.

For the KL-bow, AN-bow and TU-bow described in Tabel 1 in Kooi[5] the values, with $\overline{m}_a = 0.077 \cdot \overline{m}_b$, of the product $q \cdot \eta$ are: 0.32, 0.28 and 0.30 respectively.

For the longbow ($q \cdot \eta$=0.24) and Tartar bow ($q \cdot \eta$=0.30), see Tabel 3, the results are not in contradiction with those for the KL-bow and the TU-bow. The smaller experimental value for the longbow indicates that measured efficiency, based on the calculated $q = 0.407$, equals $\eta = 0.59$ and this is smaller than the calculated value $\eta = 0.765$. The static quality coefficient q of the TU-bow is $q = 0.491$ and this implies that the measured and calculated efficiency equal $\eta = 0.62$. The experimental data shows a relatively bad performance of the Egyptian composite bow (about $q \cdot \eta$=0.13). These results do not correlate with the results obtained with the mathematical model for the AN-bow. For the static quality coefficient q for the AN-bow we calculated $q = 0.395$ and this implies that the efficiency of the replica bow would be only $\eta = 0.33$ while the calculated value is $\eta = 0.72$.

The results for the Holmegaard bow given in Tabel 3 show that the amount of energy of the arrow per weight per draw equals $q \cdot \eta = 0.17$ and this is a rather small value. For the HO-bow we calculated in Kooi[6] that $q \cdot \eta = 0.31$. The calculated static quality coefficient equals $q = 0.364$ and this implies that the efficiency of the replica bow would be only $\eta = 0.47$ while the calculated value for the HO-bow is $\eta = 0.78$.

In the mathematical model losses due to neither damping nor hysteresis are taken into account. This implies that the calculated efficiency will generally be too high, but this cannot account for the large differences found for replica bows of the Egyptian composite bow and the Holmegaard bow because we would expect about the same effects for the replica bows of the Tartar bow and the Medieval longbow.

In Tuijn *et al.*[7] experiments showed a rather large difference in shooting from hand and from shooting-machine. Possibly for loosing with the hand, the length of the draw is not precisely defined. This effect may be larger for the Egyptian composite bow with the large draw of 101.6 cm than for the other bows with a more conventional draw of for instance 81.3 cm. This remark could also be made with respect to the bad performance of the Holmegaard bow. Alrune writes:

> "*In one movement I draw the bow to my anchor and let go without any stop*".

If this technique was also used during the experiments this could imply a certain degree of uncertainty with respect to the draw which is on the other hand assumed to be only 26 inches. Alrune reports that the bow becomes at 24 inches very heavy to draw ('stacks'). Also in this case the actual available amount of energy in the fully drawn bow is perhaps smaller than the anticipated one. So, when the actual draw at shooting was smaller than the mentioned draw this could explain part of the discrepancies.

On the other hand, when a bow stacks the amount of energy stored in the fully drawn bow per weight per draw depends sensitively on the draw. This suggests that the calculated static quality coefficient for the HO-bow which equals $q = 0.364$, is too small when the actual draw was smaller than 26 inches. Hence this counteracts somewhat the effect described above.

3.1 The kick

In Miller *et al.*[1] it is stated that:

> "*In shooting the reconstructed angular composites it was found that the central grip remains rigid throughout the draw, contributing to smooth action and greater accuracy*"

and in McEwen *et al.*[3]:

> "*Releasing the bowstring produces no kick, which results in a smooth, accurate shot*".

The results given in Tabel 3, however, do not support for the angular Egyptian bow, the statement that:

"*the composite bow is generally more efficient, so that no energy is dissipated in the kick and oscillation which characterize other bows*"

see Ref.[1]. Alrune[4] states that for the Holmegaard bow:

"*the replica is pleasant to shoot*".

Hence, we conclude that for these two bows with a bad mechanical efficiency, shooters report a pleasant bow to shoot. This is in contradiction with Klopsteg's theory. Klopsteg[9, page 170] writes:

"*The recoil, or kick, of a bow is found by experience to be small in bows of good cast, and large in sluggish, heavy bows. This is clearly a matter of efficiency. If the virtual mass is large in relation to arrow mass, the large amount of energy retained in the bow must somehow be dissipated, hence recoil becomes noticeable if not annoying*".

Klopsteg[9, page 101] states:

"*It can be said very definitely that smoothness of action and absence of kick in a bow, depend primarily on two factors. The first is dynamic balance of the limbs. ⋯ The second condition is that the bow be highly efficient, a condition somewhat depending on the first factor of dynamic balance, on the quality of wood used, and on the design of the bow. When the efficiency of the bow is high, it means that a high percentage of the energy in the limbs is transferred to the arrow, leaving very little in the bow to produce unpleasant jar or kick. A bow of low efficiency, like some steel bows I have tested, is likely to kick severely*".

On the other hand Hickman[9, page 18] mentions that a bow which:

"*bends throughout its length in the arc of a circle* (hence without rigid grip) *as a rule is not a pleasant bow to shoot because it is likely to have a unpleasant kick. The 'dip' construction (which is credited to John Buchanan of England) decreases the kick and makes a sweeter bow to shoot*".

In Ref.[6] we found that the string of a straight-end bow without a grip becomes slack after the arrow has left the bow. The large vibrating motions of the limbs cause the force in the string to become negative. When the string is suddenly stretched again it is possible that a kick is felt by the bowhand of the archer. It is tempting to claim that this explains the occurrence of a kick. If this is true then a larger internal or external damping of the material of the limbs causes a smaller efficiency but also a less severe vibrating motion of the bow and therefore probable, also after arrow exit, tension in the string. Internal friction produces heat and is called damping because it decays free vibrations of the bow so that it returns to the braced situation. Also before the arrow has left the string damping is present and this causes loss of useful energy. External damping is the friction of the limbs and arrow in the air and depends on the velocity of the subjects and this produces

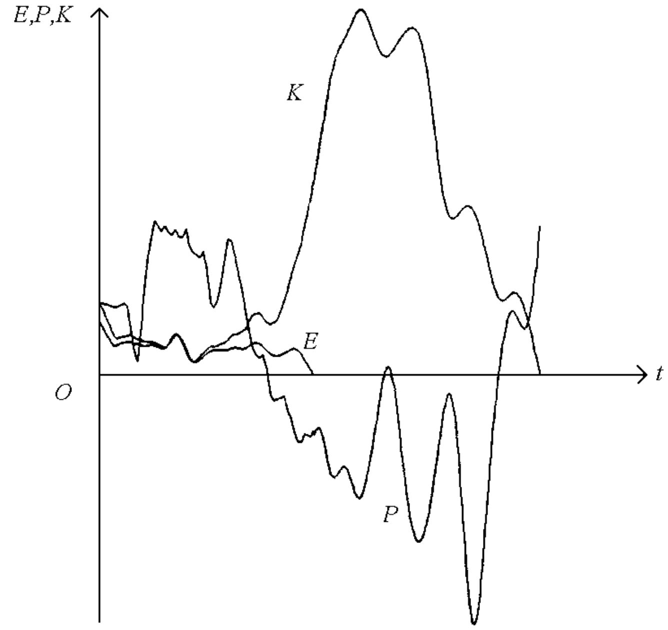

Figure 1: Calculated acceleration force E, string force K and recoil force P as function of time t for the AN-bow. The calculation is stopped when the force in the string K becomes zero.

heat too. Original energy is also dissipated partly by radiation of sound. The damping capacity of wood is higher than it is for most other structural materials. Steel is known to have a small damping capacity and thus Klopsteg's observation, that the steel bows he tested were likely to kick severely, supports our conjecture.

We conclude that the notion 'efficiency' has to be reconsidered with respect to the cause of a kick. There are two factors which contribute to loss of energy, first the virtual mass of the limbs and secondly the internal and external damping. When the efficiency is high because of small virtual mass, this means that a relatively high percentage of the energy in the limbs is transferred to the arrow, leaving very little in the bow to produce kick. When the efficiency is low because of high damping so that a large part of the energy is dissipated as heat, a relatively small amount of energy is left in the bow and this decreases the kick. This shows the influence of both factors clearly. The efficiency of the bow η defined above is the product of both factors.

In summary: a large damping for the replica of the Egyptian composite bow and of the Holmegaard flat bow explains simultaneously, a low efficiency and a pleasant bow to shoot without a kick.

But why is the internal or external damping of the materials of these replica bows much larger than for instance the damping of the materials of the Tartar bow and, perhaps to a smaller extent, of the longbow?

Otherwise the design of the angular bow and the flat bow could make them sweet bows to shoot. We calculated in Ref.[6] that the force in the string of the AN-bow becomes negative after arrow exit. In Figure 1 the force in the string together with the acceleration force E and recoil force P as functions of the time t are shown. The recoil force P is the force the bow exerts on the bowhand of the archer. For $t = 0$ the force E and the recoil force P are just the weight of the bow $\overline{F}(|\overline{OD}|)$. The results suggest that the force in the string has a rather large maximum being about 5 times as large as the weight. The recoil force shows an oscillatory behaviour after the arrow has left the string (fixed by the moment that the acceleration force becomes zero) and before the force in the string becomes zero.

4 Discussion

The cause of the bad performance of the replica bows of the Egyptian angular bow and the Holmegaard bow is still an open question. The results obtained with the mathematical model suggest that the efficiency should be in the same range as that of the Tartar composite bow and the longbow.

We encountered a number of contradicting statements about the kick. Further investigations should be done. Controlled experiments with a bow which is known to possess a kick could be performed. By introduction of artificial damping and measurement of the force in the string and the recoil force, our hypothesis, that the kick is caused by a slack string which becomes suddenly stretched, can be tested.

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